2024 NZPMC Round 2 Question Sheet

Junior

Exam Duration: 90 Minutes.

Exam Conditions: Closed book.

Materials permitted: Pen, pencil, eraser, calculator (any type).

Materials supplied to students:

1x Question and formulae booklet.

1x Answer sheet.

4x A4 refill paper (students may ask for more if required).

Instructions:

- Please write your name, year, and school on your answer sheet.
- Before the exam begins, please clear your calculators of any programs. If you do not know how to do so, raise your hand and consult one of your invigilators.
- Please do not communicate with, ask for help from or help any other participant during the exam. If you have any questions, please raise your hand and ask one of our invigilators.
- This exam consists of 20x multichoice questions (30 points total), 3x short answer questions (30 points total), and 2x harder short answer questions (30 points total). This means that there are 90 points available, ~1 point per minute.
- For multichoice questions, please fill in the corresponding circle on the answer sheet. You may cross out/rub away an answer if you wish to change it, just make sure it is clear to the markers.
- You are not expected to finish the entire exam, so don't be afraid to skip questions that you are unsure of. Good luck, and don't forget to have fun! We hope to see you back here again next year!

FORMULAE

Waves

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i} \quad \text{or} \quad s_i s_o = f^2$$

$$m=\frac{d_i}{d_o}=\frac{h_i}{h_o} \quad \text{or} \quad m=\frac{f}{s_o}=\frac{s_i}{f}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \frac{n_1}{n_2}=\frac{v_2}{v_1}=\frac{\lambda_2}{\lambda_1}$$

$$v=f\lambda \qquad f=\frac{1}{T} \qquad v=\frac{d}{t}$$

Speed of light in a vacuum $= 3.00\times10^{8}$ m s^{-1}

Mechanics

$$v=\frac{\Delta d}{\Delta t} \qquad a=\frac{\Delta v}{\Delta t} \qquad v_f = v_i + at$$

$$d=v_i t+\frac{1}{2}at^2 \qquad d=\frac{v_i+v_f}{2}t \qquad v_f^{\,2}=v_i^{\,2}+2ad$$

$$a_c=\frac{v^2}{r}$$

$$F=ma \qquad \tau = Fd \qquad F=-kx$$

$$F_c=\frac{mv^2}{r} \qquad p=mv \qquad \Delta p = F\Delta t$$

$$E_p=\frac{1}{2}kx^2 \qquad E_k=\frac{1}{2}mv^2 \qquad \Delta E_p = mg\Delta h$$

$$W=Fd \qquad P=\frac{W}{t}$$

where needed, use $g = 9.8$ m s^{-2}

Electricty

$$E=\frac{V}{d} \qquad F=Eq \qquad \Delta E_p = Eqd$$

$$E_k=\frac{1}{2}mv^2$$

$$I=\frac{q}{t} \qquad V=\frac{\Delta E}{q} \qquad V=IR$$

$$P=IV \qquad P=\frac{\Delta E}{t}$$

$$R_T=R_1+R_2+\ldots \qquad \frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}+\ldots$$

$$F=BIL \qquad F=Bqv \qquad V=BvL$$

Charge on an electron $= -1.60\times10^{-19}$ C

Mass of an electron $= 9.11\times10^{-31}$ kg

$$E(\text{thermal})=mc\Delta T \qquad E(\text{thermal})=mL$$

1. [**1 mark**] When an object is thrown vertically upward with an initial velocity, which of the following statements correctly describes its motion at the highest point of its trajectory?

   (**a**) The object's velocity is zero, and its acceleration is zero.

   (**b**) The object's velocity is zero, and its acceleration is equal to the acceleration due to gravity.

   (**c**) The object's velocity is at its maximum, and its acceleration is zero.

   (**d**) The object's velocity is at its maximum, and its acceleration is equal to the acceleration due to gravity.

2. [**1 mark**] In a simple electric circuit, a battery is connected to a resistor. Conventional current flows through the circuit. Which of the following statements best describes the direction of the conventional current?

   (**a**) The conventional current flows from the negative terminal of the battery to the positive terminal through the external circuit.

   (**b**) The conventional current flows from the positive terminal of the battery to the negative terminal through the external circuit.

   (**c**) The conventional current flows in both directions simultaneously through the circuit.

   (**d**) The conventional current does not depend on the battery terminals but on the type of resistor used.

3. [**1 mark**] Which of the following statements correctly describes the difference between transverse waves and longitudinal waves?

   (**a**) In transverse waves, the particle displacement is parallel to the direction of wave propagation, whereas in longitudinal waves, the particle displacement is perpendicular to the direction of wave propagation.

   (**b**) In transverse waves, the particle displacement is perpendicular to the direction of wave propagation, whereas in longitudinal waves, the particle displacement is parallel to the direction of wave propagation.

   (**c**) Both transverse and longitudinal waves have particle displacement perpendicular to the direction of wave propagation.

   (**d**) Both transverse and longitudinal waves have particle displacement parallel to the direction of wave propagation.

4. [**1 mark**] A car is driving on a straight, level road. The driver suddenly slams on the brakes, causing the car to come to a stop. Which of the following statements best explains what happens to the car during this process?

(**a**) The car's speed decreases to zero, and its acceleration is directed backward.

(**b**) The car's speed decreases to zero, and its acceleration is directed forward.

(**c**) The car's speed remains constant, and its acceleration is zero.

(**d**) The car's speed increases, and its acceleration is directed forward.

5. [**1 mark**] My physics teacher has a glass of water at room temperature. Warm oil is slowly poured onto the water so that it slowly builds up a layer above the water. After some time, the temperature of the water increases.

   Which of these statement(s) are necessarily true?

   1. The oil contains more thermal energy than the water initially.
   2. Heat is transferred between the oil and water through conduction.
   3. Convection does NOT occur.

(**a**) 1, and 2

(**b**) 2, and 3

(**c**) 1 only

(**d**) 2 only

6. [**1 mark**] In quantum physics, the de Broglie wavelength of an object is given by $\lambda = h/p$, where $h = 6.63 \times 10^{-34}$ Js and p is the momentum of the object. What is the debroglie wavelength of a 930 g rabbit running at a speed of 6 m/s? Note: Momentum = velocity × mass.

(**a**) 1.11×10^{-34}m

(**b**) $1.19 \times 10^{-34}m$

(**c**) $1.91 \times 10^{-34}m$

(**d**) $1.99 \times 10^{-34}m$

7. [**1 mark**] What is the circuit shown in Figure 1 below commonly referred to as?

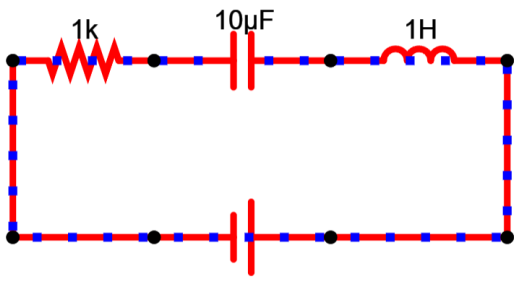


Figure 1. Circuit

(**a**) Capacitor circuit

(**b**) RLC Circuit

(**c**) RCI Circuit

(**d**) IRC Circuit

8. [**1 mark**] Which principle explains the behaviour of light when it passes from one medium to another, causing it to change direction?

(**a**) Hooke's Law

(**b**) Snell's Law

(**c**) Newton's Law

(**d**) Ohm's Law

9. [**1 mark**] A nuclear detonation experiment was conducted 8.3 light minutes away from Earth. Assuming that it was very loud and bright, how long will it take between the detonation and people on Earth to hear and see the denotation, respectively? Use values of $3.00 \times 10^8\ ms^{-1}$ for the speed of light and $340\ ms^{-1}$ for the speed of sound on Earth.

(**a**) 50 years, 8300 s.

(**b**) We won't hear it, 500 s.

(**c**) 5100 years, 8300 s.

(**d**) 5100 years, 500 s.

10. [**1 mark**] What causes the apparent weightlessness experienced by astronauts in the International Space Station?

(**a**) There is no gravity in space.
(**b**) The space station is so far from Earth that gravity is negligible.
(**c**) The space station is in free fall.
(**d**) The space station is moving so fast that gravity has no effect.

11. [**2 marks**] On a "musical road', grooves are positioned at regular intervals so that when a car drives across them, a sound wave is created. The sound's frequency is the same frequency with which a wheel hits the grooves. An engineer designs a section of a musical road to produce a sound with wavelength of 68.8 cm in air by placing the grooves 2.3 cm apart. At what speed should the cars be expected to drive in order to hear the desired note? Note: The speed of sound in air is $340\ ms^{-1}$.

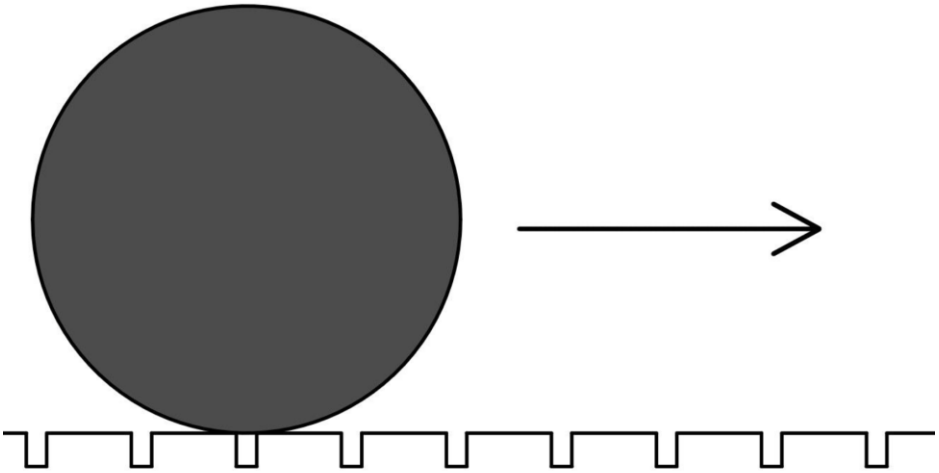


Figure 2. Diagram of wheel rolling over the musical road

(**a**) 11 kph
(**b**) 30 kph
(**c**) 41 kph
(**d**) 108 kph

12. [**2 marks**] An electric heater has a resistance of 200 ohms and is connected to a 230V power source. The heater warms a 450g of water from 25 to 55 degrees Celsius. Assuming all energy dissipated from the heater is transferred to the water, how long did this take? (Heat capacity of water is 4200J/kg/Celsius.)

(**a**) 9.5 minutes
(**b**) 5.4 minutes
(**c**) 3.6 minutes
(**d**) 2.1 minutes

13. [**2 marks**] A 58.5g tennis ball is dropped from rest very high above the ground. After falling for 460m it reaches terminal velocity, and during this time air resistance does 248.3J of work against the ball. 12 seconds after reaching terminal velocity the ball hits the ground. By considering energy changes, find the total distance the ball falls.

(**a**) 460 m
(**b**) 735 m
(**c**) 905 m
(**d**) 2060 m

14. [**2 marks**] What are the two real values of x that satisfy $3x^2 + 8x + 4 = 0$?

(**a**) $x = -\frac{2}{3}$ and $x = -2$
(**b**) $x = \frac{2}{3}$ and $x = -2$
(**c**) $x = -\frac{2}{3}$ and $x = 2$
(**d**) $x = \frac{2}{3}$ and $x = 2$

15. [**2 marks**] A regular pentagon has a side length of 8 cm, what is the distance from its centre to any one of its vertices?

(**a**) 4.11 cm
(**b**) 5.01 cm
(**c**) 5.91 cm
(**d**) 6.81 cm

16. [**2 marks**] The factorial, super-factorial and hyper-factorial are defined as the following:

$$n! = 1 \times 2 \times 3 \times \cdots \times n$$
$$n\$ = 1! \times 2! \times 3! \times \cdots \times n!$$
$$H(n) = 1^1 \times 2^2 \times 3^3 \times \cdots \times n^n.$$

Which of the following is true about $\mathrm{N} = \frac{100\$ \times \mathrm{H}(100)}{100!}$?

(**a**) N is a multiple of $10000!$.

(**b**) N is smaller than 100^{100}.

(**c**) $\sqrt[100]{N}$ is an integer.

(**d**) N has more than 10000 zeros at the end.

17. [**2 marks**] A light source at X casts a 2.1 m shadow of a vertical pole on the ground. The light source is then moved x meters upwards to the point Y, and the shadow is now 1.5 m. What is the value of x?

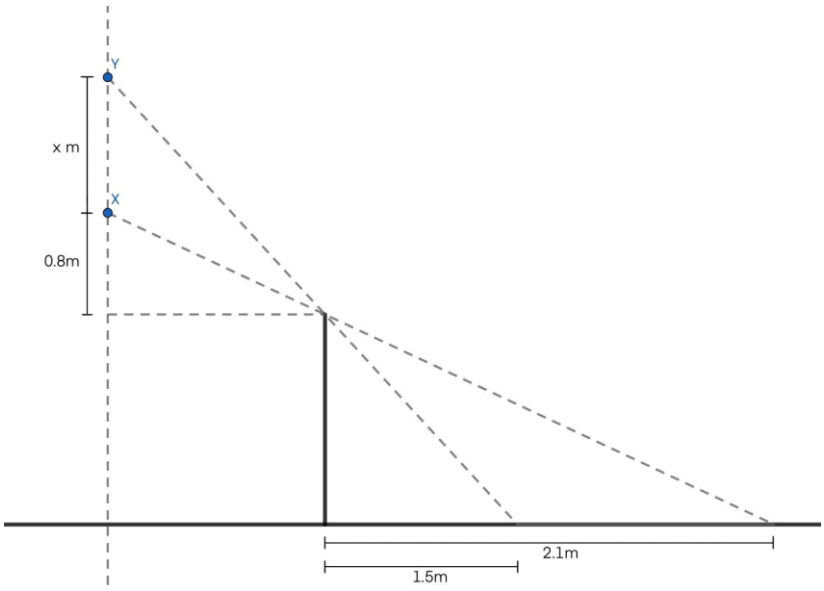


Figure 3. Diagram showing casting of the shadow

(**a**) $x = 1.28$

(**b**) $x = 1.00$

(**c**) $x = 0.64$

(**d**) $x = 0.32$

18. [**2 marks**] A circuit is shown in Figure 4, where the ratio of the voltages across the three batteries is $\varepsilon_1 : \varepsilon_2 : \varepsilon_3 = 1 : 2 : 3$, and the ratio of the resistances of the three resistors is $R_1 : R_2 : R_3 = 1 : 2 : 3$. The resistance of the wire is negligible. What is the ratio of current through the three resistors $R_1, R_2,$ $and\ R_3$?

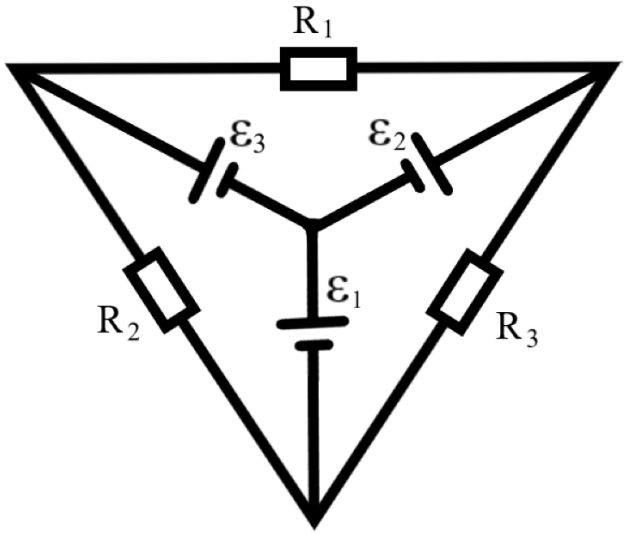


Figure 4. Three batteries and three resistors in a circuit

(**a**) 1:2:1

(**b**) 1:1:2

(**c**) 2:1:1

(**d**) 1:2:3

19. [**2 marks**] Given that the mass density of copper is about 8.9 $g\ cm^{-3}$, and the atomic mass is about 64 $g\ mol^{-1}$. The charge of an electron is $1.602 \times 10^{-19}\ C$, and Avogadro's constant is $6.02 \times 10^{23}\ mol^{-1}$. Assume that when copper conducts electricity, each copper atom contributes one free electron. When a copper wire with a circular cross-section radius of 1 mm carries a current of 4.2 A, the average drift velocity of the electrons is approximately:

(**a**) 0.01 mm/s

(**b**) 0.10 mm/s

(**c**) 1.00 mm/s

(**d**) 10.0 mm/s

20. [**2 marks**] A 200kg donkey stands in a field with all four of its hooves flat. Each hoof is circular with radius 5.00cm. What is the pressure exerted by each hoof?

(**a**) 62400 Pa
(**b**) 6240 Pa
(**c**) 15600 Pa
(**d**) 1560 Pa

21. [**10 marks**] In a grassy field, a goat is tied to the corner of a rectangular shed that measures 10 m by 12 m. The rope that leashes the goat is 15m long. To two decimal places, calculate the area of grass that the goat can eat.

22. [**10 marks**] A ball is projected vertically upwards from the ground at a speed of $5\ m\ s^{-1}$. The ball falls due to gravity, and each time it hits the ground, there's a 0.1/0.3/0.3/0.2/0.1 chance that it bounces up and at a speed that is 0%/20%/40%/60%/80% of the speed it hits the ground. The ball will bounce multiple times, if not indefinitely, but each time the distance travelled will decrease. Note: Assume $g = 10\ ms^{-2}$

(**a**) If a ball is projected up at speed u, find the total distance it travels until it reaches the ground (in terms of u). [3]

(**b**) Let $v_0 = 5$ be the initial speed of the ball going up. Let v_1 be the expected speed of the ball after the first bounce, v_2 be the expected speed of the ball after the second bounce, and so on. Using the probability distribution, find the expected value of v_n in terms of v_{n-1}.[3]

(**c**) Hence find the expected value of v_n in terms of v_0. [1]

(**d**) Using part (a) and (c), find the expected distance travelled by the ball. [3]

23. [**10 marks**] A particle of mass m moves in a circular orbit of radius, r, under the influence of a central force $F = \frac{k}{r^2}$, where k is a constant. Derive the expression for the period of the particle's orbit in terms of $m, k,$ and r. Give your answer in its simplest form.

24. [**15 marks**] A skincare company is designing a new container for their product line. The concept, illustrated in Figure 5, involves squeezing the cap's tip, which expels air from the nozzle. This expelled air is then replaced by the liquid product, allowing it to be dispensed from the container.

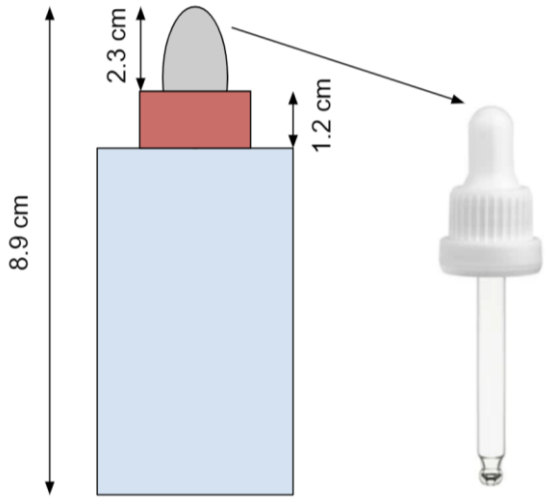


Figure 5. Diagram of squeeze container

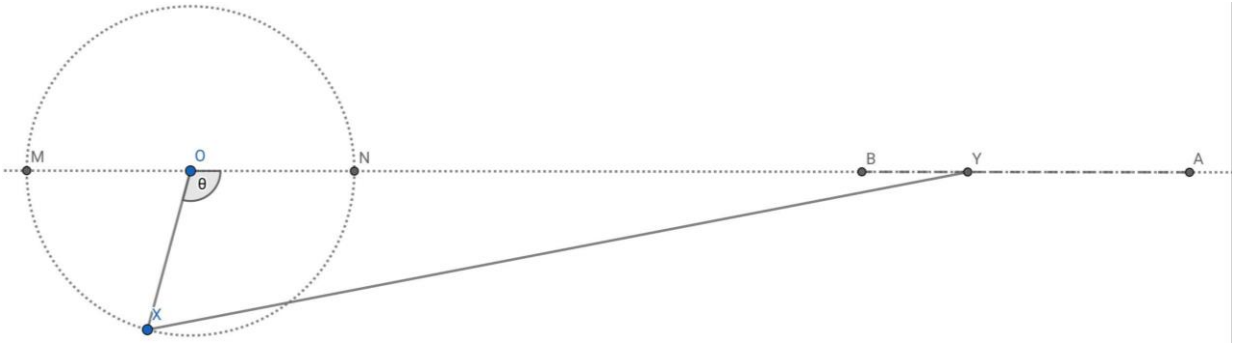
(**a**) Suppose that the glass nozzle starts from the red cap and that there should be 2mm between the bottom of the container and the end of the nozzle. How long should the glass section of the concept container be? [3]

(**b**) 2mL of liquid is inside the glass nozzle (cylinder) which has a diameter of 8mm, how high does it rise? (Assume glass has no thickness) [3]

(**c**) If one squeeze pushes out $5 \times 10^{-7}\ m^3$ of air. How many squeezes does the cap's tip need in order to get 2.1 g of product which has a density of $1{,}400\ kg\ m^{-3}$? [4]

(**d**) While testing the nozzle, you accidentally squirt a 1 g droplet of product on the floor, but you can't seem to find it. You know that you imparted 0.1 J of energy into the droplet, and it exited the nozzle horizontally (parallel to the floor). As your hand was 1.5 m above the ground when this took place, what is the maximum radius from your position that the droplet could be on the ground? [5]

25. [**15 marks**] In a model of engine piston, a rod (XY) of length l is attached to a spinning circular arm (OX) of radius r. The Y end of the rod is fixed to a horizontal axis so that when the point X rotates, the Y end of the rod oscillates back and forth between points A and B.



(**a**) By considering trigonometric identities, find a formula for the distance of OY in terms of the angle θ and constants l and r. [9]

(**b**) If the size of l increases arbitrarily compared to r, what does the function $OY(\theta)$ approach? [2]

(**c**) If the average magnitude of force on the rod at point Y acting in the direction of motion is 500 N, what is the power output of the engine at 3000 rpm in terms of r? (Note, assume the point travels back and forth at a constant speed). [4]

END OF EXAM

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