2024 NZPMC Round 2 Question Sheet

Senior

Exam Duration: 90 Minutes.

Exam Conditions: Closed book.

Materials permitted: Pen, pencil, eraser, calculator (any type).

Materials supplied to students:

1x Question and formulae booklet.

1x Answer sheet.

4x A4 refill paper (students may ask for more if required).

Instructions:

- Please write your name, year, and school on your answer sheet.
- Before the exam begins, please clear your calculators of any programs. If you do not know how to do so, raise your hand and consult one of your invigilators.
- Please do not communicate with, ask for help from or help any other participant during the exam. If you have any questions, please raise your hand and ask one of our invigilators.
- This exam consists of 20x multichoice questions (30 points total), 3x short answer questions (30 points total), and 2x harder short answer questions (30 points total). This means that there are 90 points available, ~1 point per minute.
- For multichoice questions, please circle the corresponding letter on the answer sheet. You may cross out/rub away an answer if you wish to change it, just make sure that it is clear to the markers.
- You are not expected to finish the entire exam, so don't be afraid to skip questions that you are unsure of. Good luck, and don't forget to have fun! We hope to see you back here as a competitor, or alongside us as a university student, next year!

FORMULAE

$v_f = v_i + at$

$d = v_i t + \frac{1}{2}at^2$

$d = \frac{v_i + v_f}{2}t$

$v_f^2 = v_i^2 + 2ad$

$F_g = \frac{GMm}{r^2}$

$F_c = \frac{mv^2}{r}$

$\Delta p = F\Delta t$

$\omega = 2\pi f$

$d = r\theta$

$v = r\omega$

$a = r\alpha$

$W = Fd$

$F_{net} = ma$

$p = mv$

$x_{COM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$

$\omega = \frac{\Delta\theta}{\Delta t}$

$\alpha = \frac{\Delta\omega}{\Delta t}$

$L = I\omega$

$L = mvr$

$\tau = I\alpha$

$\tau = Fr$

$E_{K(ROT)} = \frac{1}{2}I\omega^2$

$E_{K(LIN)} = \frac{1}{2}mv^2$

$\Delta E_p = mg\Delta h$

$\omega_f = \omega_i + \alpha t$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$\theta = \frac{(\omega_i + \omega_f)}{2}t$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$T = 2\pi\sqrt{\frac{l}{g}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

$E_p = \frac{1}{2}ky^2$

$F = -ky$

$a = -\omega^2 y$

$y = A\sin\omega t$	$y = A\cos\omega t$
$v = A\omega\cos\omega t$	$v = -A\omega\sin\omega t$
$a = -A\omega^2\sin\omega t$	$a = -A\omega^2\cos\omega t$

$\Delta E = Vq$

$P = VI$

$V = Ed$

$Q = CV$

$C_T = C_1 + C_2$

$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$

$E = \frac{1}{2}QV$

$C = \frac{\varepsilon_o\varepsilon_r A}{d}$

$\tau = RC$

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$

$R_T = R_1 + R_2$

$V = IR$

$F = BIL$

$V = BvL$

$\phi = BA$

$\varepsilon = -\frac{\Delta\phi}{\Delta t}$

$\varepsilon = -L\frac{\Delta I}{\Delta t}$

$\frac{N_p}{N_s} = \frac{V_p}{V_s}$

$E = \frac{1}{2}LI^2$

$\tau = \frac{L}{R}$

$I = I_{MAX}\sin\omega t$

$V = V_{MAX}\sin\omega t$

$I_{MAX} = \sqrt{2}I_{rms}$

$V_{MAX} = \sqrt{2}V_{rms}$

$X_C = \frac{1}{\omega C}$

$X_L = \omega L$

$V = IZ$

$f_0 = \frac{1}{2\pi\sqrt{LC}}$

$v = f\lambda$

$f = \frac{1}{T}$

$n\lambda = \frac{dx}{L}$

$n\lambda = d\sin\theta$

$f' = f\frac{V_W}{V_W \pm V_S}$

$E = hf$

$hf = \phi + E_K$

$E = \Delta mc^2$

$\frac{1}{\lambda} = R\left(\frac{1}{S^2} - \frac{1}{L^2}\right)$

$E_n = -\frac{hcR}{n^2}$

Useful data

Speed of light $= c = 3.00\times10^8$ m s^{-1}

Charge on an electron $= q = -1.60\times10^{-19}$ C

Acceleration due to gravity on Earth $= g = 9.81$ m s^{-2}

Permittivity of free space $= \varepsilon_0 = 8.85\times10^{-12}$ F m^{-1}

Universal Gravitational Constant $= G = 6.67\times10^{-11}$ N m^2 kg^{-2}

1. [**1 mark**] A car's braking system can be modelled as a spring followed by a hydraulic mechanism between the pedal and a brake pad (pushing against a brake disk) as shown in figure 1. In this model, the velocity of the brake pedal relative to the car is directly proportional to what? Note: Assume the hydraulic fluid is incompressible, and movement of the brake pad is negligible.

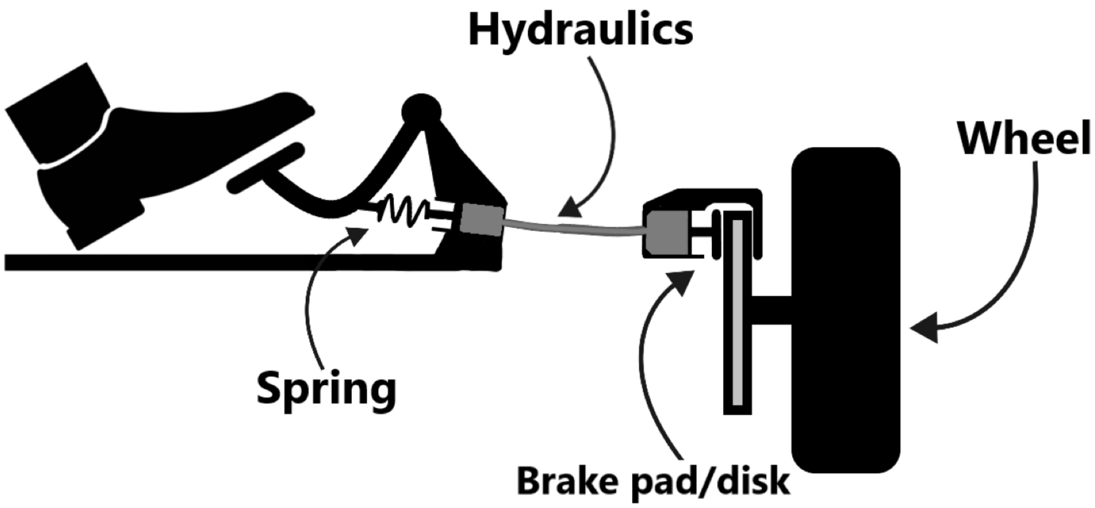


Figure 1. A diagram of brake system model.

(**a**) The car's velocity.

(**b**) The car's acceleration.

(**c**) The car's jerk.

(**d**) The car's snap.

2. [**1 mark**] Two astronauts, Joanna and Bob, have parked their spaceship a safe distance from a black hole. They both start 20-hour timers on their watches, before Joanna slowly sets off toward the black hole (relative velocity is negligible). Joanna uses her jet pack to approach the black hole and then returns to the ship. When she enters the air lock, Bob's alarm goes off. Which of the following statements is true?

(**a**) Joanna's alarm has already gone off.

(**b**) Joanna's alarm hasn't gone off yet.

(**c**) Joanna's alarm goes off at the same time as Bob's.

(**d**) Joanna's timer still has 20 hours left.

3. [**1 mark**] A truck and a car are on a frictionless surface. The truck is initially moving at $10\ ms^{-1}$ and collides with the car which begins at rest. After the collision, the truck and car move as a single unit at $5\ ms^{-1}$. Which of the following statements about the above scenario is correct?

(**a**) The collision is elastic because both momentum and kinetic energy are conserved
(**b**) The collision is inelastic because the cars stick together, indicating kinetic energy is not conserved.
(**c**) The collision is elastic because the total momentum is conserved.
(**d**) The collision is inelastic because momentum is not conserved.

4. [**1 mark**] Find the equation of the tangent line to the curve $y = \sqrt{x}$ at the point where $x = 4$.

(**a**) $y = \frac{1}{4}x + 1$
(**b**) $y = 4x + \frac{1}{4}$
(**c**) $y = -\frac{1}{4}x + 1$
(**d**) $y = -4x + \frac{1}{4}$

5. [**1 mark**] A 1.5-metre-long string is fixed at both ends, and supports a standing wave with 3 nodes (including the endpoints). What is the wavelength of the standing wave?

(**a**) $0.50\ m$
(**b**) $0.75\ m$
(**c**) $1.50\ m$
(**d**) $3.00\ m$

6. [**1 mark**] Which particle is responsible for mediating the electromagnetic force?

(**a**) The electron
(**b**) The electromagnetron
(**c**) The photon
(**d**) The gluon

7. [**1 mark**] Which of the following describe Kirchhoff's laws? (select all that apply)

(**a**) The total current entering a junction equals the total current leaving the junction.
(**b**) The power dissipated in a circuit is equal to the product of current and voltage.
(**c**) The resistance in a circuit is equal to the sum of individual resistances.
(**d**) The total voltage around any closed loop is equal to the sum of the voltage drops within the same loop.

8. [**1 mark**] You are in an extremely positively charged spaceship, and are about to zoom over the magnetic north pole of a planet. With the north pole of the planet directly beneath you, in which direction does the Lorentz Force act on your ship?

(**a**) Upwards, away from the magnet.
(**b**) Downwards, toward the magnet.
(**c**) To your left.
(**d**) To your right.

9. [**1 mark**] In an AC circuit, what is the phase relationship between the voltage and the current in a purely capacitive circuit?

(**a**) Voltage leads the current by 90 degrees.
(**b**) Current leads the voltage by 90 degrees.
(**c**) Voltage and current are in phase.
(**d**) Voltage and current are out of phase by 180 degrees.

10. [**1 mark**] In classical electrodynamics, the Larmor formula lets us calculate the total power radiated by a charged and accelerating particle due to electromagnetic radiation. It expresses this power P in terms of the particle's charge q and acceleration a, along with the constants $\pi, \varepsilon_0\ and\ c$.

$$P = \frac{2}{3}\frac{q^2a^2}{4\pi\varepsilon_0 c^3}$$

If the particle's charge is halved but its acceleration is tripled, by what factor does the radiated power change?

(**a**) $\frac{9}{4}$

(**b**) $\frac{3}{2}$

(**c**) $\frac{1}{2}$

(**d**) $\frac{3}{4}$

11. [**2 marks**] If a diver is 10 m under water ($1000\ kg/m^3$), and the water is under 8 km of air ($1.29\ kg/m^3$), what is the total pressure on the diver, relative to a vacuum?

(**a**) $199\ kPa$

(**b**) $200\ kPa$

(**c**) $101\ kPa$

(**d**) $100\ kPa$

12. [**2 marks**] Shion pushes her brother Luka on a swing. His displacement from equilibrium, x (m), is given by the equation $x = 3\ e^{-\frac{t}{2}} \cos{(5t)}$. What is the amplitude of Lukas' motion after four full swings?

(**a**) $1.61\ m$

(**b**) $0.858\ m$

(**c**) $0.458\ m$

(**d**) $0.244\ m$

13. [**2 marks**] Given that $\sum_{r=0}^{N}\binom{N}{r} = 2^N$, find the value of

$$\binom{2024}{0}\times 2024 + \binom{2024}{1}\times 2023 + \ldots + \binom{2024}{2024}\times 0$$

(**a**) 2024×2^{2023}
(**b**) 2023×2^{2023}
(**c**) 2023×2^{2024}
(**d**) 2024×2^{2024}

14. [**2 marks**] Let k be a real number such that $s = 6 + ki$ and $t = 4 + ki$ are complex numbers that satisfy $arg(st) = \frac{\pi}{4}$. Find the value of k.

(**a**) $k = -12$
(**b**) $k = 2$
(**c**) $k = 12$
(**d**) $k = 6$

15. [**2 marks**] Alice and Bob race in their spaceships to see who can cover 105,000 km first. Alice travels 500 km on the first day, and each subsequent day she travels 500 km more than the previous day (i.e., she has travelled a total of 1,500 km after the second day). Each day, Bob travels 25% less than the distance travelled the previous day. What is the minimum distance Bob must travel on the first day to at least tie with Alice?

(**a**) 16,500 km
(**b**) 24,300 km
(**c**) 26,300 km
(**d**) 36,500 km

16. [**2 marks**] William won a trip to Mars onboard a spaceship that rotates to simulate gravity for its crew. William brought his trusty bathroom scales with him, but when he used them, they only read 40 kg! If William's room is 120 m from the axis of rotation, and completes one revolution every 30 seconds, what would his scales read if he weighed himself back on Earth?

(**a**) 40 kg
(**b**) 65 kg
(**c**) 70 kg
(**d**) 75 kg

17. [**2 marks**] A straight wire is positioned at the origin of a 2D plane and perpendicular to the page. A constant stream of electrons flows through the wire into the page. The magnetic field pattern caused by the current on the page can be expressed as a function $f(z)$; the input z is a complex number representing a point in the plane, and the output $f(z)$ represents the magnetic flux density, both magnitude and direction, at that point. Which of the following is a possible expression for $f(z)$? Note: c is a positive constant and $\bar{z}$ denotes the complex conjugate of z.

(**a**) $f(z) = -c\frac{i}{\bar{z}}$
(**b**) $f(z) = c\frac{i}{\bar{z}}$
(**c**) $f(z) = -c\frac{iz}{(z\bar{z})^{\frac{3}{2}}}$
(**d**) $f(z) = c\frac{iz}{(z\bar{z})^{\frac{3}{2}}}$

18. [**2 marks**] A 40g mass is placed onto a pump bottle, shown in figure 2, which has a spring constant of $20\ Nm^{-1}$. A small amount of substance is launched out at an initial velocity of $7\ ms^{-1}$. If the bottle stands at 30 cm tall unpumped, which of the evenly spaced markers (A, B, C, or D) does the substance land closest to?

Note: Assume that the weight force of the 40g mass is the force used to compress the spring.

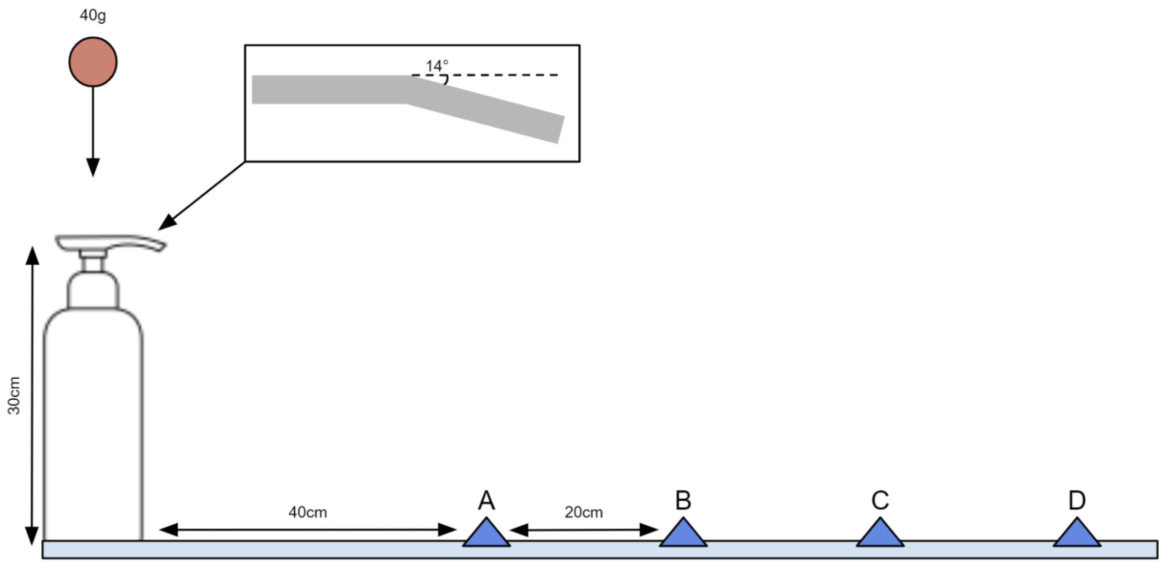


Figure 2. Bottle placed before evenly spaced markers.

(**a**) A

(**b**) B

(**c**) C

(**d**) D

19. [**2 marks**] The quantity, x (g), of a substance changes with time, t (s), according to the following differential equation:

$$\frac{dx}{dt} = -kx^{2024}$$

where k is a positive constant. The half-life of the substance is first measured at time $t = 0\ s$ then at time $t = 1\ s$. How much longer is the second half life compared to the first one?

(**a**) $1 + 2 + 4 + \cdots + 2^{2022}\ s$
(**b**) $1 + 2 + 4 + \cdots + 2^{2023}\ s$
(**c**) $1 + 2 + 4 + \cdots + 2^{2024}\ s$
(**d**) $1 + 2 + 4 + \cdots + 2^{2025}\ s$

20. [**2 marks**] (NOT MULTICHOICE)

The arc length of a curve is the distance between two points along that curve. If $y = f(x)$ is a real function that is continuously differentiable, then the arc length between $x = a$ and $x = b$ is given by

$$\text{Arc length} = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

An astroid curve, shown in figure 3, is a star-like figure that can be expressed in Cartesian coordinates as

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$$

Find the arc length of the astroid curve. (Hint: you may like to rearrange $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ for y and then observe the symmetry of the astroid curve).

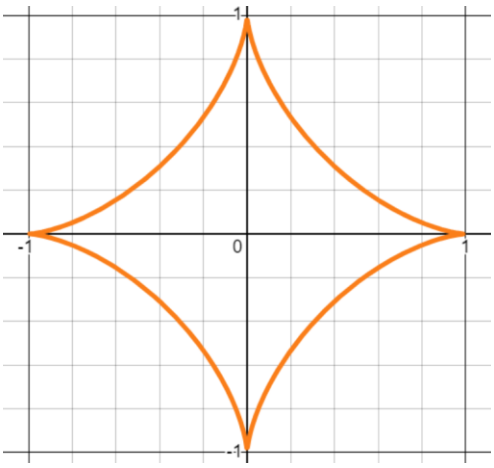


Figure 3. Astroid Curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$.

21. [**10 marks**] A manometer is a tool used to determine pressure differential between two points. Traditional manometers used a U-shaped tube filled with mercury, the difference in the height of the mercury on each side reveals the difference in pressure. Two water pipes at different heights and pressures are shown in Figure 4 below. Use the levels of the manometer to find the pressure of pipe B. Note: The pressure of each pipe is measured at the height that the manometer tube connects to it.

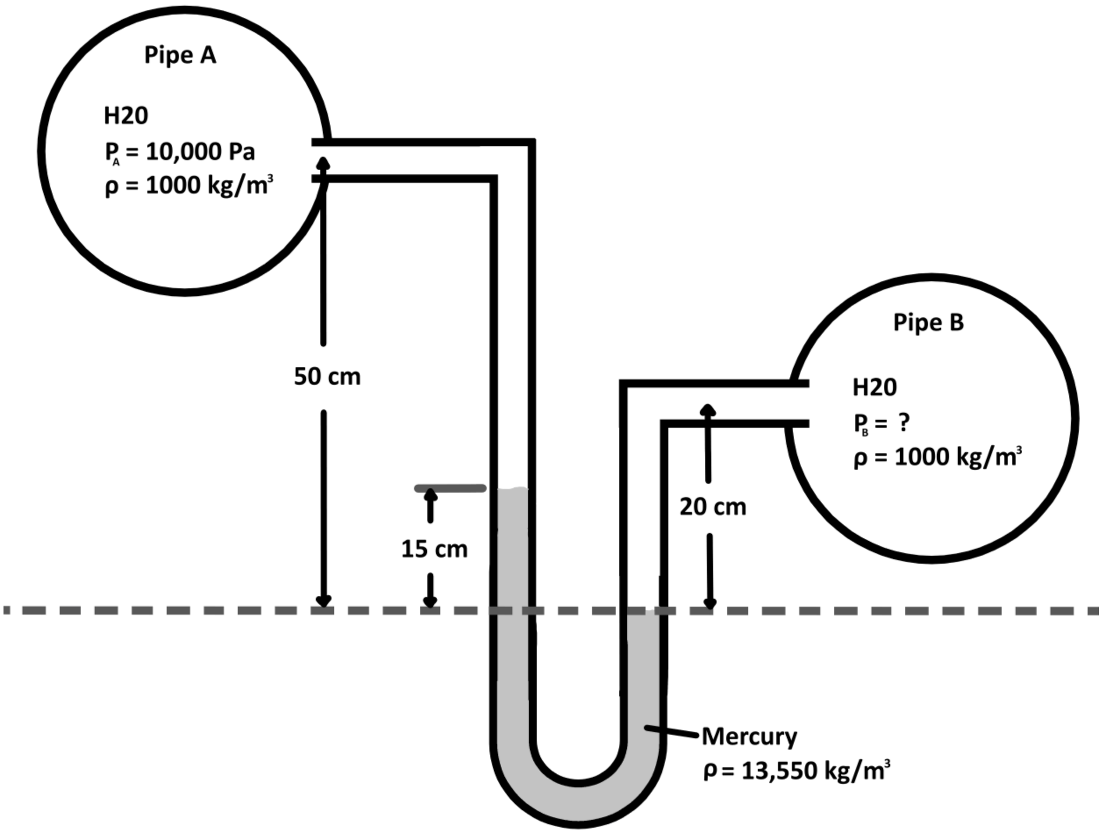


Figure 4: Cross section of pipes A and B, with a mercury manometer between them.

22. [**10 marks**] An AC voltage source with frequency f is applied to an inductor with inductance L and a resistor with resistance R in series.

Give, in terms of f, L, and R:

(**a**) The magnitude of the impedance of the resistor [1]

(**b**) The magnitude of the impedance of the inductor [3]

(**c**) The magnitude of the total impedance of the circuit [5]

(**d**) The average proportion of energy dissipated across the resistor [1]

23. [**10 marks**] Ray chops a 1.0 m long straight rod at two random points. What is the probability that he can form a triangle with the resulting three pieces? Note: To be able to form a triangle, the sum of any two sides must be greater than the third.

24. [**15 marks**] Hard-clipping is a process which "deletes" parts of an AC voltage above a certain threshold a replaces them with the threshold value. The following wave has period $T = 2\pi$, and original peak voltage of 12 V, but has since been clipped off at 10 V.

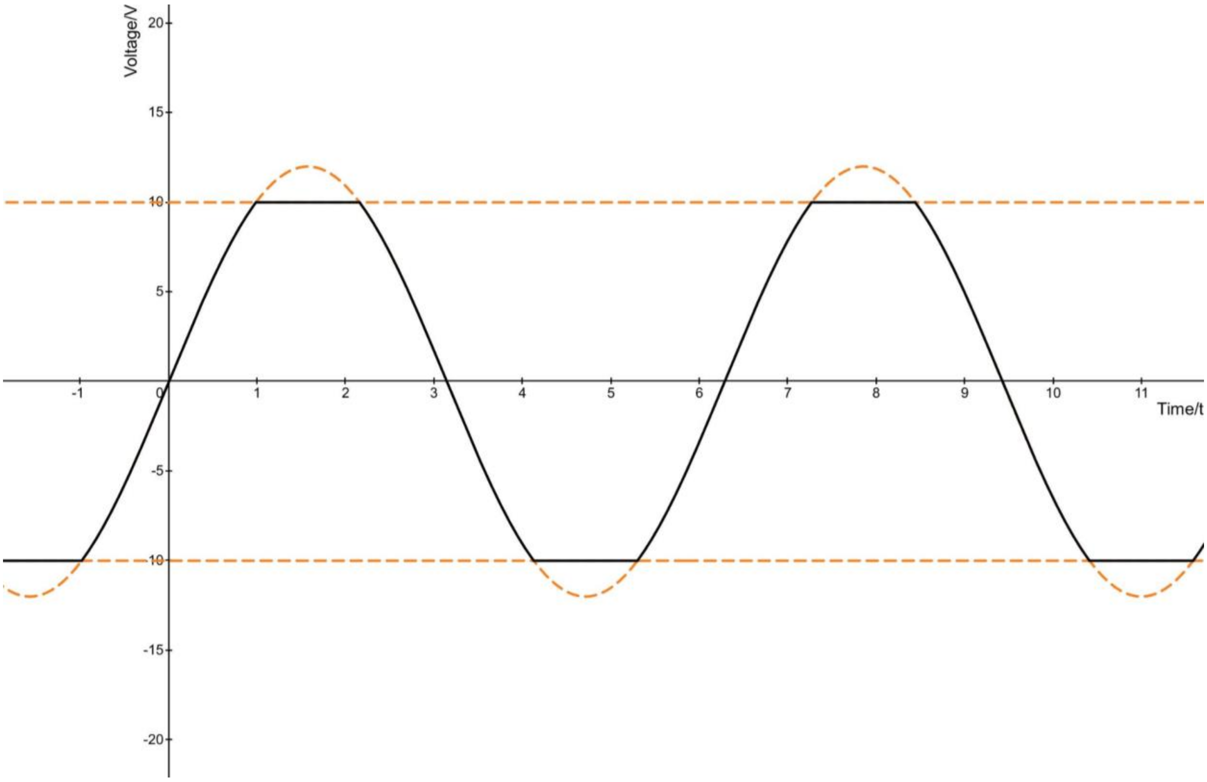


Figure 5. Hard-clipped AC voltage.

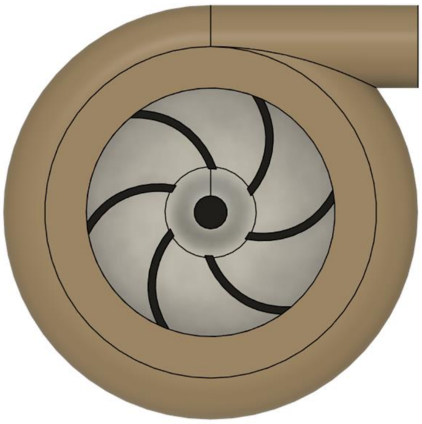
(**a**) In a circuit with constant resistance $R = 5\ \Omega$, what is the relationship between power and voltage? [2]

(**b**) Describe a piecewise function which represents the power dissipated over time for $0 \leq t \leq \pi$. [3]

(**c**) By considering its definition, find the RMS voltage of this signal. [5]

(**d**) What percentage of power is lost due to this clipping? (Consider the power dissipated without the clipping.) [5]

25. [**15 marks**]



Your friend secretly signed you up for an engineering position at Rocket Blab with a fake CV. You got the job and now it's your first day, your task is to design the impeller blades for the liquid Oxygen (LOX) pump in their new rocket engine. An impeller or centrifugal pump takes in a fluid axially, and uses spinning blades to flick the water outwards (radially). For this rocket engine, you have been instructed to use a semi open impeller design, shown in Figures 6 and 7. To make blade design simpler, we can describe the blade in terms of $\varphi(R)$, where R is the radial distance from the centre of the axle, and φ is the angle of the blade at that distance.

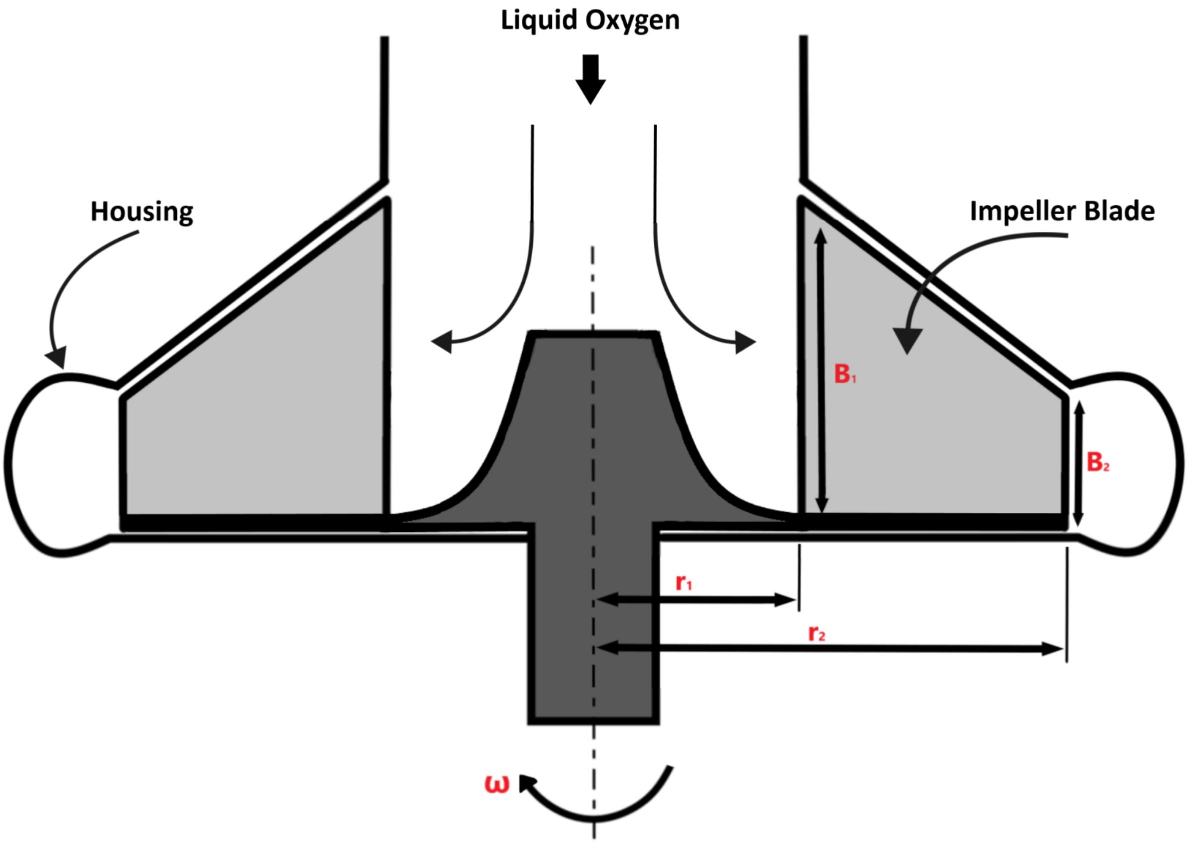


Figure 6. Cross section of LOX pump, with dimensions of one blade labelled.

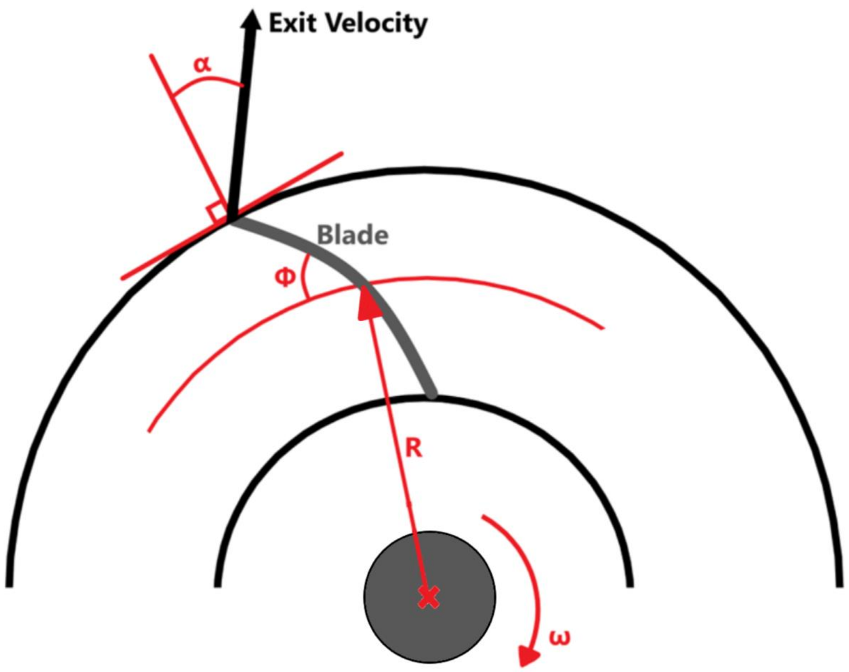


Figure 7. Top-down view of a single blade in the impeller pump, showing relevant variables.

(**a**) If the volumetric flow rate, $V_{Flow\ Rate}$, of LOX is 0.3 $m^3\ s^{-1}$, and $B_1 = 5\ cm$, what is the radial (outwards) velocity, V_1, of the LOX at R $= r_1 = 7$ cm? Note: It can be assumed that the blades are infinitely thin. Hint: Imagine the flow being evenly distributed through the walls of a cylinder with dimensions $r = r_1$, $h = B_1$. [2]

(**b**) If the rotational velocity, ω, is $100\ rad\ s^{-1}$, what must the angle of each blade, φ, be at R $= r_1 = 7$ cm? Note: It can be assumed that the flow is everywhere tangent to the blade surface when viewed from a reference frame rotating with the blade, and that at R $= r_1$ the flow is purely radial. [5]

(**c**) Determine the shape of a blade between R $= r_1$ and R $= r_2$ as a function $\varphi(R)$, with constants $V_{Flow\ Rate},\ \omega,\ r_1,\ r_2,\ B_1,\ and\ B_2$. Noting that the height of the blades decreases linearly with radius, from B_1 to B_2. Note: Assume that the flow is purely radial ($\alpha = 0°$). [3]

(**d**) For an impeller where $V_{\text{Flow Rate}} = 0.3\ m^3 s^{-1},\ \omega = 100\ rad\ s^{-1},\ r_1 = 7$ cm, $r_2 = 15$ cm, $B_1 = 5$ cm, and $B_2 = 2$ cm. What is the power required for an exit angle of $\alpha = 20°$? Note: The density of liquid oxygen, $\rho = 1140\ kg\ m^{-3}$. [5]

END OF EXAM

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